Mechanical Problem Set #2 - FACT and Screw Motions

Executive Summary:

A screw-motion flexure was designed to exhibit two distinct pitches – 1 and 3 mm/deg nominally – by having an interchangeable cross bar linkage with different cross-sectional dimensions and axial lengths. Functional Requirements (FRs) specifying translational and rotational stiffnesses about the x-axis (see Appendix A for coordinate system and FRs) were met by devising an analytical model that considers the stiffness contributions of all 5 members in the flexure topology, assuming they were undergoing fixed-guided or pinned moment bending. After testing using a force gauge, angle gauge, and calipers, two screw motions with pitches that differed by a ratio of 2.67 over a range of 3° was observed.

Math Model: [Link to Spreadsheet]

a) b)

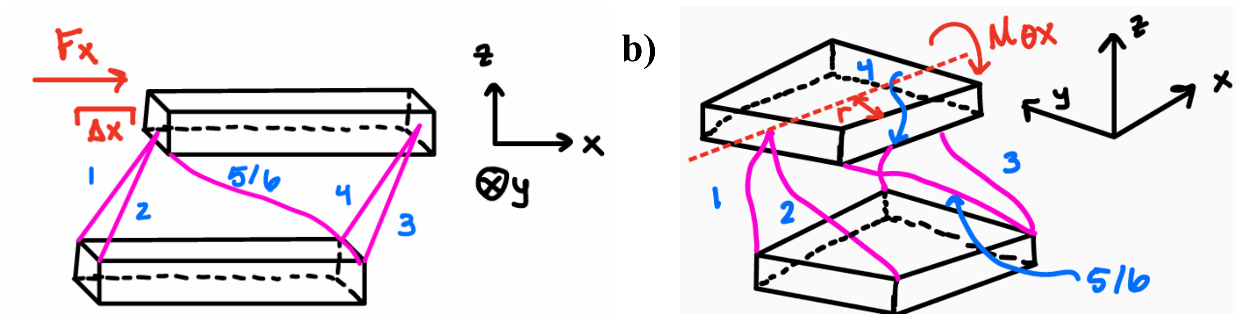


Figure 1. Translational (a) and rotational (b) motions about the x-axis. **(a)** All links are undergoing fixed-guided bending **(b)** Links 1-4 are undergoing pinned-moment bending and link 5/6 is undergoing fixed-guided bending.

Although translation and rotation about x is coupled, a unique stiffness for both lateral translation (Fig. 1a) and rotation (Fig. 1b) were calculated independently. To calculate the assembly's translational stiffness, we used a fixed-guided beam bending model for all trusses, modeling Truss 5/6 as bending in the x-direction. To calculate rotational stiffness we used pinned-moment beam bending models for Trusses 1-4, and modeled Truss 5/6 as fixed-guided beam bending in the z-direction acting as a moment a distance *r* from the x-axis of rotation. To determine the pitch of our mechanism for each configuration, we applied the pitch equation $p = r * tan(\theta_2)$. We used θ_2 in case of truss 5, and θ_3 for truss 6. Based on this equation, only the *r* and θ_2 dimensions impact the pitch, and not the stiffness of any of the trusses.

All other assembly stiffnesses corresponding to constrained DOFs (translation and rotation about y and z) were calculated summing the stiffness contributions from members undergoing axial tension and compression. For every other stiffness, axial tension and compression is the most significant stiffness contribution, allowing us to simplify our model by negating stiffness contributions from fixed-guided beam bending or pinned-moment beam bending.

Verification Method & Results:

Our verification setup consisted of a force gauge, angle gauge, and calipers. We measured the displacement using the calipers which have a resolution of 0.01 mm. The displacement was measured for a given force, which we applied through the force gauge which has a resolution of 0.1 N. Upon applying this force, we were able to measure the rotation angle of the screw motion via the angle gauge which has a resolution of 0.1 degrees.

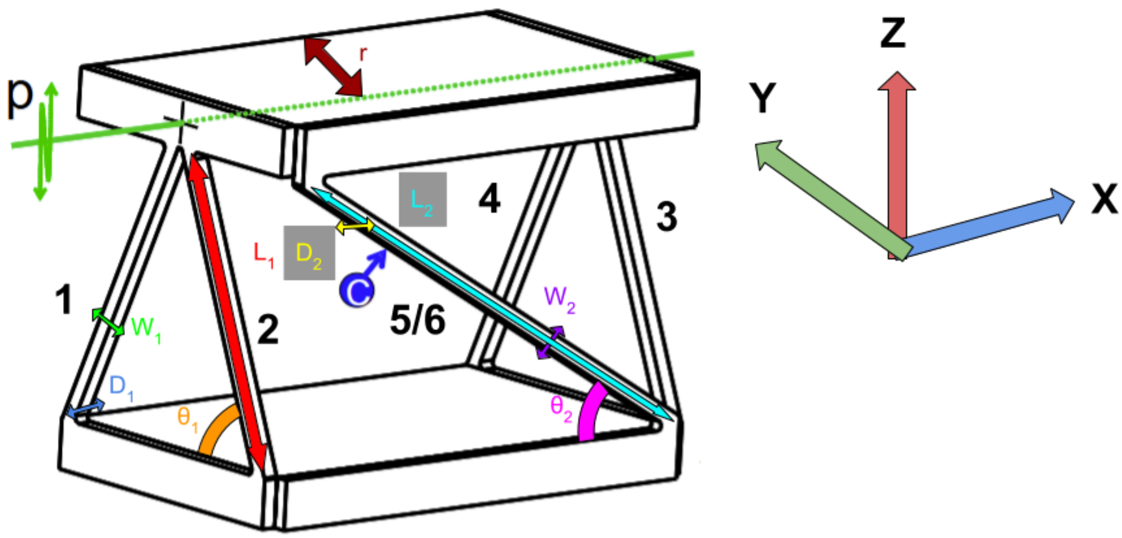
Our verification setup, consisting of the force gauge, angle gauge, and calipers.

Discussion:

Initially, we were fairly confident that the analytical model used for calculating assembly stiffnesses in MPS #1 was directly applicable for calculating the translational and rotational stiffnesses about the x-axis for this MPS. Therefore, our first analysis strategy entailed only considering the stiffness contributions from members that were experiencing axial tension/compression. For example, to calculate translational stiffness in x, we only considered the stiffness contribution of the cross bar because we believed it was the only link undergoing axial compression. However, this resulted in an excessively high calculated translational stiffness that did not match the stiffness that was experimentally determined by testing a corresponding wood prototype. Our current analysis method considers the stiffness contributions of all 5 members in the assembly, assuming they are undergoing bending instead. The strategy employed for MPS #1 likely worked because all 6 DOFs were constrained, so only considering stiffness contributions from axial loading was sufficient; for MPS #2, however, our device needed to permit motions about the x-axis which was achieved by bending all 5 links. Had axial loading dominated, that would've effectively introduced an unintentional constraint. Thus, we ended up only considering stiffness contributions from links in axial loading for calculating stiffnesses in directions that we expected to be constrained.

When designing our flexure system, we understood that the pitch depends on the geometry of the system (r and θ_2), but the dimensions of the flexure are what control stiffness and therefore enable a discernible pitch. Therefore, we calculated stiffnesses in K_X and $K_{\theta X}$ such that our desired pitches could be observed by a person pushing on the system. Testing a wood prototype, we measured the ratio of pitches and determined this to be within specifications, while the K_X and $K_{\theta X}$ stiffnesses were off from our calculated values. We then fine-tuned our flexure dimensions slightly to obtain K_X and $K_{\theta X}$ stiffnesses within our functional requirements, while maintaining the desired ratio of pitches. From this MPS, we learned that our models will not always be completely accurate due to considerations made for laser cutting such as avoiding stress concentrations using fillets. However, models still provide an extremely important basis on which to fine-tune adjustments to meet requirements.

Appendix A



Variable Name	Symbol	Range	Justification	Validation
Smaller Pitch	P_S	1 ± 0.2 mm/deg $0.8 < P_S < 1.2$ mm/deg	1 is big enough to provide a pitch that can be seen over 3 degrees. 0.2 mm uncertainty due to the resolution of the human eye and it can still be measured via calipers.	2.9 mm over 3 degrees = **0.96 mm/deg**
Larger Pitch	P_L	3 ± 0.6 mm/deg	x3 of the smaller pitch (uncertainty scaled proportionally)	7.68 mm over 3 degrees = **2.56 mm/deg**
Ratio of Larger to Smaller Pitch	$R_{L,S}$	3 $2.45 < R_{L,S} < 3.67$	Calculated via the biggest and smallest differences in pitches. $\frac{3.3\ mm}{0.9\ mm} = 3.67$ $\frac{2.7\ mm}{1.1\ mm} = 2.45$ Uncertainty due to clearance variation.	2.56 mm/deg / 0.96 mm/deg = **2.67**
Displacement of diagonal flexure in X for larger pitch	δ_{XL}	9 ± 1.8 mm	Flexure should move this amount based on the larger pitch = (3 ± 0.6 mm/deg) * 3 deg	**7.68 mm**

Displacement of diagonal flexure in X for smaller pitch	δ_{XS}	3 ± 0.6 mm	Flexure should move this amount based on the smaller pitch = (1 ± 0.2 mm/deg) * 3 deg	**2.9 mm**
Lateral Stiffness in X (Larger Pitch)	k_{XL}	1.67 N/mm 1.38 N/mm < K_{XL} < 2.083 N/mm	An average human could comfortably exert 15 N with a finger (~3.4lbs) to observe a screw motion with a net displacement of 9 mm. K_{XL} = 15N/(3 mm/deg*3 deg) Range factors in P_L uncertainty.	**1.93 N/mm**
Lateral Stiffness in X (Smaller Pitch)	k_{XS}	5 N/mm 4.16 N/mm < K_{XS} < 12.5 N/mm	Same reasoning as above, but to observe displacement of 3 mm. K_{XS} = 15 N/(3 mm/deg*3 deg) Range factors in P_S uncertainty.	**9.03 N/mm**
Rotational Stiffness in X	$k_{\theta X}$	0.3125 ± 0.0625 Nm/deg	Stiffness such that 15 N of force allows for 3 deg of rotation. $K_{\theta X}$ = 15 N * r mm / 3 deg r = 62.5 ± 12.5 mm Lower bound for r determined by ergonomic dimensions for top and bottom plate.	15 N achieved 1.8 degrees of rotation for r = 42.5 mm (force was applied at edge). This leads to **0.354 Nm/deg**
Minimum Lateral Stiffness in Y and Z to Lateral Stiffness in X	k_Y , k_Z	75 N/mm	15 N / 0.2 mm = 75 N/mm 0.2 mm due to the resolution of the human eye	k_Y = 15 N / 0.08 mm = **187.5 N/mm** k_Z = 48 N / 0.06mm = **800 N/mm**
Minimum Rotational Stiffness in Y and Z	$k_{\theta Y}$, $k_{\theta Z}$	6.66 Nm	20 Nm/3 deg = 6.66 Nm Since 20 Nm is the max wrist flexion torque, X direction.	117 N * 0.155/2 m = 1 degree for y $k_{\theta Y}$ **= 9.07 Nm/deg** 27.8 N * 0.155/2 m = 0.3 degrees for z $k_{\theta Z}$ **= 7.18 Nm/deg**

Max Allowable Stress of Flexures	σ	10.7 MPa	The yield stress of acrylic is 75 MPa. Using a safety factor of 7, calculated as a human can exert 105 N of force with their arm, and 15 N with their finger, the max stress the acrylic is allowed to endure is 10.7 MPa	**10.08 MPa**
Height of Device	h	42 mm to 292.1 mm	In order to ensure that the top and bottom platforms have enough clearance when they rotate 3 deg, $\tan(3\ \text{deg}) = \frac{h}{75\ mm}$ $\rightarrow$ h = 3.93 mm h + height of polypropylene = 3.93 mm + 2(19.05 mm) = 42 mm 11.5 in to mm = 292.1 mm In order to fit in a backpack and be transportable	**178.86 mm**
Mass	m	0.5 lbs < m < 5 lbs	Our last MPS was 0.8 lbs and we don't want it to be so heavy that we can't carry it around and so it's not too hard to handle while using it.	Measured using a scale with resolution of 0.1 g. Weighed 541.1 g = **1.19 lbs**